June 24, 2002

Calibrated Economic Models Add Focus, Accuracy, and Persuasiveness to Merger Analysis

Gregory J. Werden
Luke M. Froeb[*]

The traditional competitive analysis of mergers was developed mainly by judges in the United States, with training in neither economics nor antitrust, who had to decide whether particular mergers substantially lessened competition. Economists participated in that process mainly as expert witnesses, typically offering little more than ultimate conclusions. Economic models (for example, models of oligopoly) and empirical studies (for example, of the relationship between market concentration and price) were at most a basis for crude intuition about the effects of increased market concentration.

Traditional merger analysis has been giving way to a more scientific inquiry that applies the full panoply of tools provided by modern economics. The competitive analysis of mergers increasingly employs formal microeconomic models and econometrics—statistical analysis designed for, and applied to, economic data.[1] Of particular significance in merger cases is the use of "calibrated economic models," i.e., quantitative analysis using formal economic models in which the values of the key parameters are based on the observable facts of the merger under review. The calibration of models to the facts of the case may be based on econometric studies or direct measurements of relevant quantities. And calibrated economic models may be used to inform the traditional structural analysis of mergers, based on market delineation and market shares, or used instead of structural analysis.

---

[*] Werden is Senior Economic Counsel, Antitrust Division, U.S. Department of Justice; gregory.werden@usdoj.gov. The views expressed herein are not purported to reflect those of the U.S. Department of Justice. Froeb is William C. and Margaret M. Oehmig Associate Professor of Entrepreneurship and Free Enterprise, Owen Graduate School of Management, Vanderbilt University; luke.froeb@owen.vanderbilt.edu.

[1] For a survey of uses of econometrics in antitrust litigation, see Jonathan B. Baker & Daniel L. Rubinfeld, *Empirical Methods in Antitrust Litigation: Review and Critique*, 1 AM. L. & ECON. REV. 386 (1999).

Calibrated economic models offer three advantages in merger analysis. First, they bring key issues into sharper focus by making assumptions explicit and identifying which factors are critical and precisely how they matter. Second, they add accuracy to the analysis by quantifying issues of importance and relying on calculations rather than intuition. Third, they make the analysis more persuasive in a judicial proceeding by making it more concrete and better grounded in both the facts of case and economic theory.[2] These advantages are illustrated below first in market delineation and second in directly assessing the competitive effects of mergers.

Calibrated Economic Models for Market Delineation

In traditional structural analysis, market delineation is central to horizontal merger cases, and it often has proved decisive in court. It is not surprising, therefore, that calibrated economic models are most commonly used, and have been most influential, in market delineation. The hypothetical monopolist paradigm for market delineation, which has become a standard tool for merger analysis around the world,[3] holds that a collection of products and an area constitute a market only if a hypothetical monopolist over them would maximize its profits by raising price at least some specific threshold amount, such as 5%. This paradigm is now commonly implemented using calibrated economic models.

The formal economic model of monopoly is both very simple and relatively straightforward to apply to market delineation using the hypothetical monopolist paradigm. The model teaches that the monopolist sets its price to equate its price-cost margin (price minus marginal cost, all divided by price) with the reciprocal of its elasticity

[2] This third advantage may be more important in the United States than in some other places. In the United States, only the courts have the power to enjoin the consummation of a merger. In the European Union, and some countries, competition authorities can prevent consummation of a merger, although their decisions can be overruled by courts.

[3] *See* Gregory J. Werden, The 1982 Merger Guidelines and the Ascent of the Hypothetical Monopolist Paradigm, *available at* http://www.usdoj.gov/atr/hmerger/11256.htm. For details of the application of the paradigm, see Gregory J. Werden, *Demand Elasticities in Antitrust Analysis*, 66 ANTITRUST L.J. 363, 387–96 (1998); Gregory J. Werden, *Market Delineation under the Merger Guidelines: A Tenth Anniversary Retrospective*, 38 ANTITRUST BULL. 517 (1993); Gregory J. Werden, *Market Delineation and the Justice Department's Merger Guidelines*, 1983 DUKE L.J. 514; Gregory J. Werden, Market Delineation Algorithms Based on the Hypothetical Monopolist Paradigm (unpublished paper April 22, 2002).

of demand.[4] The main difficulty in applying this lesson is that the monopolist's elasticity of demand depends on its price. Demand generally is more elastic at higher prices, and the monopoly price normally exceeds the pre-merger price at which the elasticity of demand is assessed in market delineation. To make direct use of the monopoly model, it is therefore necessary to make an assumption about how the elasticity of demand changes with price, or equivalently, about the curvature of demand. A common and relatively conservative assumption is that demand is linear.[5]

Assuming linear demand, it is straightforward to operationalize the hypothetical monopolist test in terms of a "critical elasticity of demand" or "critical sales loss."[6] The former is the highest pre-merger elasticity of demand the hypothetical monopolist could face and still want to raise price at least the threshold amount. The latter is the maximum loss in unit sales the hypothetical monopolist would be willing to suffer and still raise price at least the threshold amount.[7] The critical elasticity of demand and the critical sales

---

[4] Demand elasticity is the responsiveness of the quantity consumers demand to a change in price. Responsiveness of a product's quantity to its own price is an "own elasticity of demand," and responsiveness to the price of another product is a "cross elasticity of demand." Both are expressed as a quotient; the numerator being the percentage change in quantity, and the denominator being the percentage change in price inducing that quantity change. The greater the own elasticity of demand, the more "elastic" demand is said to be. Demand is also said to be "elastic" ("inelastic") when the own elasticity of demand is less than 1, meaning that a 1% change in a product's price induces more (less) than a 1% reduction in its quantity demanded.

[5] Only linear demand and constant elasticity demand yield simple formulas as in footnote 8. From the perspective of a plaintiff challenging a merger, linear demand normally is the more conservative assumption, as it makes it more difficult to pass the hypothetical monopolist test.

[6] There is an extensive literature on these tools: Michael G. Baumann & Paul E. Godek, *Could and Would Understood: Critical Elasticities and the Merger Guidelines*, 40 ANTITRUST BULL. 885 (1995); Kenneth Danger & H.E. Frech III, *Critical Thinking About "Critical Loss" in Antitrust*, 46 ANTITRUST BULL. 339 (2001); Barry C. Harris & Joseph J. Simons, *Focusing Market Definition: How Much Substitution Is Enough*, *in* 12 RESEARCH IN LAW AND ECONOMICS 207 (Richard O. Zerbe, Jr. ed., 1989); Frederick I. Johnson, *Market Definition under the Merger Guidelines: Critical Demand Elasticities*, *in* 12 RESEARCH IN LAW AND ECONOMICS 235 (Richard O. Zerbe, Jr. ed., 1989); James Langenfeld & Wenqing Li, *Critical Loss Analysis in Evaluating Mergers*, 46 ANTITRUST BULL. 299 (2001); Werden, *Demand Elasticities*, *supra* note 3, at 410–11; Gregory J. Werden, *Four Suggestions on Market Delineation*, 37 ANTITRUST BULL. 107, 119–20 (1992).

[7] Described in the text is the "profit-maximization critical loss," which is consistent with the profit-maximization assumption in the hypothetical monopolist paradigm. More commonly used

loss are entirely determined by the price-increase threshold (typically specified to be 5%) and the pre-merger price-cost margin.[8]

The hypothetical monopolist test is routinely calibrated from accounting data reflecting the industry price-cost margin:[9] Having measured that margin, it is immediately clear when the demand faced by the hypothetical monopolist is so elastic that it would not raise price at least the threshold amount. If the margin is quite high (80–100%), the critical elasticity of demand is close to 1, meaning that a loss in sales of only about 5% would be sufficient to dissuade the hypothetical monopolist from increasing price by 5%. If the margin is quite low (less than 25%), the critical elasticity of demand is greater than 3, meaning that a loss in sales of more than 15% would be necessary to dissuade the hypothetical monopolist from increasing price by 5%. Typical margins (50–60%) yield critical demand elasticities of roughly 1.5.

Critical elasticity and critical loss analysis bring market delineation into much sharper focus. It has long been understood that market delineation is about demand elasticity, but critical elasticity and critical loss analysis make it exquisitely clear that the only relevant demand elasticity is the own elasticity of demand faced by the hypothetical monopolist. Most importantly, critical elasticity and critical loss analysis indicate exactly when the hypothetical monopolist's demand would be sufficiently inelastic to induce the hypothetical monopolist to raise price significantly. Finally, critical elasticity and critical loss analysis highlight the importance of pre-merger price-cost margins as a determinant of the relevant market.

---

is the "breakeven critical loss"—the greatest reduction in quantity the hypothetical monopolist could experience and still not suffer a net loss in profit from the threshold price increase. The attractive feature of breakeven critical loss is that it does not depend on the functional form (curvature) of demand. And if the price-increase-significance threshold is small, like 5%, and the margin high, the breakeven critical loss is quite close to the profit-maximization critical loss.

[8] Denoting the price-increase threshold as t and the price-cost margin as m (both expressed as proportions), the critical demand elasticity with linear demand is $1/(m + 2t)$, and the critical sales loss is $t/(m + 2t)$. The breakeven critical loss for any demand curve is $t/(m + t)$. *See* Werden, *Demand Elasticities*, *supra* note 3, at 388–91, 410–12.

[9] Calibration can be subtle: There may be significant conceptual issues in reckoning the relevant marginal cost, and the larger the reduction in output and the longer the period of time allowed to adjust to the post-merger environment, the greater the associated reduction in cost and the lower the relevant price-cost margin.

Because formal economic models are built on explicit assumptions, they also focus the analysis by facilitating an inquiry into how well they "fit" the facts of a case.[10] The proper use of calibrated economic models involves careful consideration of the facts and constructs models consistent with them. Standard critical elasticity and critical loss calculations assume, for example, that the hypothetical monopolist has constant marginal costs. When this assumption is unrealistic, the standard calculations should not be used; rather, a more realistic cost model can be calibrated. If investigation reveals that different units of productive capacity have differing marginal costs, the hypothetical monopolist can be modeled accordingly.[11] The additional information required to calibrate the hypothetical monopolist's cost function is not difficult to obtain in many cases.

Most often, a calculated critical elasticity or critical loss is used as a yardstick to evaluate the significance of non-quantitative evidence on likely consumer switching in the event of a price increase. In such cases, critical elasticity and critical loss analysis enhance the accuracy of merger analysis by providing a concrete basis for evaluating qualitative evidence on substitution, indicating, for example, whether a little substitution is enough to defeat a price increase.[12] Significantly greater accuracy is achieved by combining such analyses with econometric estimation of the relevant demand elasticity.

---

[10] In the United States, expert testimony may be excluded for lack of "fit" with the facts of the case. *See* General Electric Co. v. Joiner, 522 U.S. 136, 146 (1997) ("A court may conclude that there is simply too great an analytical gap between the data and the opinion proffered."); Brooke Group Ltd. v. Brown & Williamson Tobacco Corp., 509 U.S. 209, 242 (1993) ("When expert testimony is not supported by sufficient facts to validate it in the eyes of the law, or when indisputable record facts contradict or otherwise render the opinion unreasonable, it cannot support a jury's verdict. Expert testimony is useful as a guide to interpreting market facts, but it is not a substitute for them.") (citation omitted).

[11] Similarly, it is easy to model the scenario in which quasi-fixed costs are avoided as output is decreased because some productive capacity is shut down. And it is straightforward to model more complex demand scenarios, for example, a product with several distinct uses and significantly different elasticities of demand in the different uses.

[12] Critical loss analysis was used in this way and was highly significant in several litigated merger cases in the United States. FTC v. Tenet Health Care Corp., 186 F.3d 1045, 1050–51, 1053 (8th Cir. 1999); United States v. Mercy Health Services, 902 F. Supp. 968, 980–81 (N.D. Iowa 1995), *vacated as moot*, 107 F.3d 632 (8th Cir. 1997); California v. Sutter Health System, 84 F. Supp. 2d 1057, 1076–80 (N.D. Cal. 2000), *aff'd*, 217 F.3d 846 (9th Cir. 2000), *opinion amended by* 130 F. Supp. 2d 1109, 1128–32 (N.D. Cal. 2001).

While not a recent development, the use of estimated demand elasticities in market delineation has now become fairly common, in part because the development of critical elasticity analysis has provided a useful guide to their interpretation.[13]

Econometric evidence on demand elasticities is most needed, most helpful, and most often used with differentiated consumer products. With such products, documents and interviews tend to leave critical questions of degree unanswered. Unlike producer goods for which substitution issues are apt to turn on objective cost issues, with differentiated consumer goods, such issues inherently are matters of taste. Econometric evidence often is the most accurate, reliable, and objective basis for evaluating critical questions of degree involving consumer tastes, because they tend to be too idiosyncratic to be reckoned with sufficient accuracy without data on actual choices or survey responses. Critical elasticity analysis greatly enhances the accuracy of econometrics-based market delineation by providing a specific value with which to compare the estimated elasticity for a candidate market.[14]

Critical elasticity and critical loss analysis enhance the persuasiveness of a market delineation argument to a court, and in the United States, that is especially important for the government, which has the burden of proof. The burden is significant because district court judges have tended to be skeptical of the anticompetitive effects of challenged mergers.[15] Especially over the last decade, judges have been inclined to find that a little substitutability is sufficient to place products in a relevant market. The problems the

---

[13] A quarter century ago, the government's expert estimated the elasticity of demand for frozen dessert pies in an effort to show that they constituted a relevant market. The court found his "testimony completely useless, primarily because we have no basis for evaluating what a particular elasticity coefficient means." United States v. Mrs. Smith's Pie Co., 440 F. Supp. 220, 227–28 (E.D. Pa. 1976). For further discussion, see Gregory J. Werden, *Simulating the Effects of Differentiated Products Mergers: A Practical Alternative to Structural Merger Policy*, 5 GEO. MASON L. REV. 363, 371 (1997).

[14] A United States court was first presented with (but did not rely on) a critical elasticity of demand analysis supported by econometric demand estimates in *FTC v. Swedish Match Co.*, 131 F. Supp. 2d 151, 160–61 (D.D.C. 2000).

[15] In the 1960s, the United States Supreme Court did not share this skepticism and ruled in the government's favor in many merger cases. The Supreme Court would not have had the opportunity to do so had the government not lost so frequently in district court. At that time, the Justice Department appealed directly to the Supreme Court.

government confronts, and the potential of calibrated economic models to solve them, are illustrated by two Department of Justice merger cases.

The merger case most recently tried by the Department concerned "disaster recovery" services for computer systems.[16] The facts were complicated because different businesses have different "recovery time objectives" and different computer facilities. The combination of a complex factual setting and the extraordinary pace of litigation[17] made it difficult for the Department to carry its burden on market delineation, and the court found the Department failed to do so.[18]

The Department alleged that the relevant market was "shared hotsite services" (for certain types of computer equipment), which provide a relatively rapid recovery time, at a relatively low cost, by serving multiple clients with the same computer facilities. The central issue in the case was whether alternatives, especially internally provided hotsites, potentially providing even more rapid recovery, were in the relevant market. The court found that the government had shown that *some* customers would *not* switch away from shared hotsite services in response to a 5% price increase, but failed to show that the number of such consumers was "substantial enough that a hypothetical monopolist would find it profitable to impose such an increase in price."[19]

The defendants presented the court with a critical loss analysis purporting to show that the critical loss was only 5% because margins were extremely high.[20] Because the Department offered no contrary critical loss analysis,[21] the defendants' analysis stood

---

[16] United States v. SunGard Data Systems, Inc., 172 F. Supp. 2d 172 (D.D.C. 2001).

[17] The court rendered its decision just 24 days after the complaint in the case was filed. This extraordinary pace resulted from pending bankruptcy proceedings.

[18] 172 F. Supp. 2d at 182, 186–92.

[19] *Id.* at 191–92.

[20] *Id.* at 190 n.21.

[21] The defendants' treated all costs associated with computer hardware and software as fixed, resulting in a marginal cost that was a tiny fraction of price. This would make perfect sense if the hardware and software were long lived assets while the service was sold on a short-term basis. In this industry, however, hardware and software were replaced fairly frequently and services were sold though long-term contracts. Thus, it might have been argued that the relevant price-cost margin actually was quite low.

uncontested, and we suspect that the defendants' analysis substantially influenced the way the court viewed the substitutability evidence. That analysis indicated that very little substitution was enough to defeat a price increase, and it was impossible for the Department to show that even such little substitution would not occur. Had the court been presented with an analysis indicating that a great deal of substitution was required to prevent a price increase, the showing made by the Department might have been viewed as sufficient to establish the alleged relevant market.

Also illustrative is the 1995 challenge to the merger of leading bakers of branded white bread.[22] Like the vast majority of government merger cases in the United States, it was settled by a consent decree providing for the divestiture of assets (in this case, principally brands). Had the case gone to trial, the defendants most likely would have argued that other types of bread, perhaps all sources of carbohydrates, were in the relevant market. There is no doubt that these other products are substitutes for white bread, and the court likely would have been skeptical about a relevant market limited to white bread. In support of its relevant market, the Department's expert calculated critical demand elasticities (for different local markets) and compared them to demand elasticities estimated from supermarket scanner data. This evidence indicated, with very high statistical confidence, that demand was less elastic than the critical value.[23] The government's presentation surely would have been more persuasive to a skeptical judge because it used quantitative evidence in a concrete manner to shed light on the difficult questions of degree presented by market delineation.

Critical elasticity and critical loss analysis is routinely used, and it is highly influential. It has been said in litigation that "some number beats no number." The reason is that the introduction of any respectable quantitative analysis is apt to control the debate, and thereby likely win it. The use of calibrated economic models also means that expert testimony is no longer a black box to the court. Properly chosen and carefully

---

[22] United States v. Interstate Bakeries Corp., No. 95C-4194 (N.D. Ill., filed July 20, 1995); 60 Fed. Reg. 40,195 (Aug. 7, 1995) (hold separate stipulation and competitive impact statement).

[23] The Department's expert was an author of this paper. A highly condensed version of his expert report in the case (which was not filed at the time) was subsequently published. For his analysis of the relevant market, see Gregory J. Werden, *Expert Report in* United States v. Interstate Bakeries Corp. and Continental Baking Co., 7 INT'L J. ECON. BUS. 139, 141–43 (2000).

calibrated economic models provide direct, scientific connections between the facts of a case and the ultimate conclusions reached.[24]

Calibrated Economic Models for Predicting Competitive Effects

Economics offers a variety of models that may be employed in a variety of ways to assess the competitive effects of mergers. Just as the monopoly model can be used to analyze the price or quantity set by a hypothetical monopolist, oligopoly models can be used to analyze how a merger affects the prices or quantities set by merging firms and their rivals. But no oligopoly model capturing the complexities of a real-world competitive process also is simple enough to permit calibration based on observable data or simple enough to yield useful predictions. The art of modeling is simplifying reality in a manner that captures what is important for the purposes of the analysis to be undertaken. An appropriate model in any particular case reflects both the significance of individual competitors and the essence of the competitive process in the industry.[25]

For a merger involving highly differentiated consumer products, the model must account for brand preferences in reasonably realistic manner. If firms compete primarily on the basis of price, at least in the relatively short-term, price should be the strategic decision variable for competitors. For a merger involving a homogeneous product and

---

[24] This connection is essential in the United States, because expert conclusions lacking a scientific foundation are entitled to no weight. *See, e.g.*, Kumho Tire Co. v. Carmichael, 526 U.S. 137, 152 (1999) (an expert must "employ[] in the courtroom the same level of intellectual rigor that characterizes the practice of an expert in the relevant field"); SMS Systems Maintenance Services, Inc. v. Digital Equipment Corp., 188 F.3d 11, 25 (1st Cir. 1999) ("Expert testimony that offers only a bare conclusion is insufficient to prove the expert's point."); Mid-State Fertilizer Co. v. Exchange National Bank of Chicago, 877 F.2d 1333, 1339 (7th Cir. 1989) ("An expert who supplies nothing but a bottom line supplies nothing of value to the judicial process.").

[25] For example, the competitive interaction in many industries closely resembles an auction, and formal auction models have been used to predict the competitive effects of mergers in such industries. *See* Lance Brannman & Luke M. Froeb, *Mergers, Cartels, Set-Asides, and Bidding Preferences in Asymmetric Oral Auctions*, 82 REV. ECON. & STAT. 283 (2000); Serdar Dalkir, John Logan & Robert T. Masson, *Mergers in Symmetric and Asymmetric Noncooperative Auction Markets: The Effects on Prices and Efficiency*, 18 INT'L J. IND. ORG. 383 (2000); Steven Tschantz, Philip Crooke & Luke Froeb, *Mergers in Sealed versus Oral Auctions*, 7 INT'L J. ECON. BUS. 201 (2000); Luke M. Froeb & Steven Tschantz, *Mergers Among Bidders with Correlated Values*, *in* MEASURING MARKET POWER (Daniel J. Slottje, ed. forthcoming 2002).

competitors distinguished by their costs and production capacities, the model must account for costs and capacities in a reasonably realistic manner. If a single market price is determined by aggregate quantity competitors make available, quantity should be the strategic decision variable for competitors.

Oligopoly models are "equilibrium" models, i.e, they determine a set of competitive strategies (usually prices or quantities) at which no competitor has an incentive to change its strategy, given the strategies of rivals.[26] Calibrating such a model involves setting its parameters so that it exactly predicts the pre-merger equilibrium. For example, plugging the pre-merger prices into the model must yield the pre-merger shares.[27]

The simplest oligopoly model for predicting the competitive effects of mergers probably is the "dominant firm model."[28] It posits that all firms but one in an industry act as a "competitive fringe," producing up to the point at which their marginal costs of production equal the market price, as all firms do in a competitive industry. The remaining firm is the dominant firm, and it acts as a monopolist with respect its "residual demand curve," i.e., the portion of total industry demand that the competitive fringe does not supply. This model may be appropriate in an industry with a homogeneous product if the merged firm would be substantially larger than its rivals. The model can be calibrated from information on the elasticity of market demand and the pre-merger margins and productive capacities of the relevant competitors.[29]

---

[26] This is Nash, non-cooperative equilibrium, formalized by mathematician John F. Nash, who shared the 1994 Nobel Memorial Prize in Economics for this work.

[27] Calibration requires that a set of prices and shares be deemed the pre-merger equilibrium. Prices and shares averaged over a recent period generally are used; however, the prices and shares used may never have been observed but rather are thought to be likely in the near future if the merger does not occur. While we refer to the benchmark model as "pre-merger," it is meant to reflect the world that will prevail but for the merger. Failing to calibrate in this manner is a common error that renders meaningless the comparison between the predicted prices post merger and the actual prices pre merger.

[28] This model is credited to Karl Forchheimer, *Theoretishes zum unvollständigen Monopole*, 32 JAHRBUCH FÜR GESETZGEBUNG, VERWALTUNG UND VOLKSWIRTSCHAFT 1 (1908). It is the model used by William M. Landes & Richard A. Posner, *Market Power in Antitrust Cases*, 94 HARV. L. REV. 937 (1981).

[29] Such an analysis was employed by the Department of Justice in its analysis of Georgia Pacific Corp.'s acquisition of Fort James Corp., which was challenged on the basis of likely

The use of calibrated economic models for predicting the competitive effects of mergers is referred to as "merger simulation," and we have found it especially well suited to the analysis of the competitive effects of mergers involving differentiated consumer products.[30] The standard oligopoly model applied to such products is the Bertrand model,[31] which assumes that price is the only short-term strategic variable through which competition occurs. A Bertrand equilibrium is a set of prices such that each competitor is happy with its price given those of rivals.

One reason we find the Bertrand model well suited to predicting the effects of differentiated products mergers is that it accurately reflects what a merger does in such an industry. A merger mainly internalizes the competition between formerly separately owned brands, and that is precisely what occurs in the model. And to the extent that merger synergies reduce production cost, that is easily incorporated. While price is never the only strategic variable in the real world, we have often concluded that little violence to reality is done by considering only price competition.[32] We have also generally found

---

anticompetitive effects on away-from-home tissue products. *See* United States v. Georgia Pacific Corp., No. 00-2824 (D.D.C., filed Nov. 21, 2000); 66 Fed. Reg. 9,096 (Feb. 6, 2001) (complaint and competitive impact statement).

[30] To date, there has been very little courtroom use of merger simulation in the United States. One reason for this is that the two federal enforcement agencies each have only about one merger trial per year. We know little of the details but understand that an analysis similar to that we advocate was unsuccessfully used by the plaintiffs in *New York v. Kraft General Foods, Inc.*, 926 F. Supp. 321 (S.D.N.Y. 1995), a merger case involving ready-to-eat breakfast cereals. An academic analysis prompted largely by that case is provided by Aviv Nevo, *Mergers with Differentiated Products: The Case of the Ready-to-Eat Cereal Industry*, 31 RAND J. ECON. 395 (2000). Analyses prepared for use in litigation are provided by Jerry A. Hausman & Gregory K. Leonard, *Economic Analysis of Differentiated Products Mergers Using Real World Data*, 5 GEO. MASON L. REV. 321 (1997) (analyzing a tissue merger); and Werden, *supra* note 23, at 144–46 (analyzing a bread merger).

[31] The Bertrand model is named for Joseph Louis François Bertrand and stems from a book review he published in 1883. A modern translation by James W. Friedman appears at COURNOT OLIGOPOLY 73 (Andrew F. Daugherty ed., 1988). For a general proof that mergers without efficiencies raise prices in Bertrand models of differentiated products, see Raymond Deneckere & Carl Davidson, *Incentives to Form Coalitions with Bertrand Competition*, 16 RAND J. ECON. 473 (1985).

[32] Various sorts of promotions, such as sales, are important marketing strategies for many highly differentiated consumer products. The simulation model omits this sort of marketing, for example, summarizing a complicated schedule of prices by a single average price, but we

that the intensity of competition in the Bertrand model matches well with that observed pre merger.[33]

A differentiated products merger simulation is calibrated with readily observable information on the prices and "shares" of brands in the simulation, and with potentially observable information on the elasticities of demand (own and cross) of those brands.[34] A simulation may be more or less inclusive than the relevant market, and these "shares" are merely the relative quantities for the included brands.

The prices of brands included in a simulation are determined by the competition among them and their prices may change as the merger alters competition. The prices of brands excluded from a simulation are assumed to be unaffected by the merger. Narrowing the list of included brands merely narrows the list of brands for which prices may increase, and because the prices of excluded products generally would be affected very little by a merger, excluding them just imparts a very slight downward bias to the price increase predictions. The critical implication of the foregoing is that market

---

generally do not view this simplification as problematic. Similarly, consumers often choose among a large number of configurations of a particular product, while the model generally reflects them as a single brand aggregate, but we do not believe this is a serious problem.

[33] To predict the competitive effects of mergers, it is necessary to "recover" the marginal costs for each product in the model. This normally is *not* done by directly measuring costs. Rather, the equilibrium conditions of the model are solved for the marginal costs implied by the observed prices and shares. The implied marginal costs then can be compared with cost information that may be available. We generally have found that the implied marginal costs correspond closely to what is known about actual marginal costs, at least for major products. What that means is that the markup of price over cost in the model, and hence the intensity of competition, is at least roughly the same as the intensity of actual competition.

[34] For a concise statement of the process of Bertrand merger simulation with differentiated consumer products, see Gregory J. Werden, *Simulating Unilateral Competitive Effects from Differentiated Products Mergers*, ANTITRUST, Spring 1997, at 27. More complete statements of the analysis are found in Philip Crooke, Luke M. Froeb, Steven Tschantz & Gregory J. Werden, *The Effects of Assumed Demand Form on Simulated Postmerger Equilibria*, 15 REV. INDUS. ORG. 205 (1999); Werden, *supra* note 13; Gregory J. Werden, *Simulating the Effects of Differentiated Products Mergers: A Practitioners' Guide*, *in* STRATEGY AND POLICY IN THE FOOD SYSTEM: EMERGING ISSUES 95 (Julie A. Caswell & Ronald W. Cotterill eds., 1997); Gregory J. Werden & Luke M. Froeb, *Simulation as an Alternative to Structural Merger Policy in Differentiated Products Industries*, *in* THE ECONOMICS OF THE ANTITRUST PROCESS 65 (Malcolm B. Coate & Andrew N. Kleit eds., 1996).

delineation is irrelevant to merger analysis based on merger simulation.[35]

Ideally, the demand elasticities used to calibrate the model would be estimated from a rich data source that makes it possible to reliably measure all of the relevant own and cross elasticities of demand. Precisely estimated demand elasticities significantly enhance the fit between the model and the facts of the case and hence significantly increase the accuracy of the predictions as well as the persuasiveness of the analysis in court. The data available in the real word, however, are never ideal and generally present a trade-off between variance and bias.[36]

The number of elasticities that must be estimated increases with the square of the number of brands included in a simulation. Unless some structure is imposed on substitution patterns, their number easily may be so large that the data are inadequate to the task. Econometricians then say that the estimator has a high "variance."[37] Variance can be reduced by asking less of the data, which is done by imposing structure on substitution patterns, but that may mean imposing unrealistic substitution patterns. Econometricians then say that the estimator is "biased."

At one extreme in the variance-bias trade-off is the logit model, in which just two parameters determine all of the own and cross elasticities of demand for the included brands.[38] One of these parameters is the aggregate elasticity of demand for all brands in

---

[35] In the United States, case law precedent mandates market delineation, but we believe it is only a matter of time before courts embrace direct methods for predicting the competitive effects of mergers, since they already embrace direct evidence of market power. We also believe that delineating a relevant market actually may undermine a challenge to a merger. With highly differentiated consumer products, the relevant market delineated by the hypothetical monopolist paradigm may be as narrow as the two merging brands (if they are next-best substitutes and the merger would increase their prices at least 5%), yet such narrow markets are not alleged in merger complaints because of a well-founded belief that judges would reject them out of hand. And when a broad relevant market is alleged, some judges take this to be a concession that all products in the market are very close substitutes for each other.

[36] Estimating demand elasticities with real-world data presents a host of complex issues beyond the scope of this paper.

[37] A common symptom of high variance is negative estimated cross elasticities, indicating brands are complements, even though they are known to be substitutes.

[38] We refer mainly to the Antitrust Logit Model (ALM), a reformulation of the conventional logit model designed to make it more user friendly to practitioners of merger analysis. For details

the simulation, and it plays basically the same role in merger simulation that market delineation plays in traditional structural analysis. If the demand for the included brands is sufficiently elastic, excluded brands are sufficiently close substitutes for the included brands that mergers of included brands cannot increase prices significantly. The greater the value of the second demand parameter, the greater the substitutability among included brands. If this parameter is very low, the included brands are such distant substitutes for each other that each is essentially a monopoly unto itself, so the merger of two included brands has little effect on their prices. If this parameter is very high, the included brands are such close substitutes for each other that only a merger to monopoly among the included brands could have much effect on their prices.

The logit model forces substitution patterns to exhibit the Independence of Irrelevant Alternatives (IIA) property.[39] In practical terms, this means that substitution from any brand to all others is proportionate to their relative shares. If brands *A*, *B*, and *C* have shares of 60%, 30%, and 10%, and the price of brand *C* is increased, the IIA property says that the substitution from brand *C* to brand *A* must be twice that from *C* to *B*, because the share of *A* is twice that of *B*.

Absent contrary evidence, substitution in proportion is often viewed as the most natural default assumption.[40] We share that view because we think the IIA property most

---

of the model, see Werden & Froeb, *supra* note 34; Gregory J. Werden, Luke M. Froeb & Timothy J. Tardiff, *The Use of the Logit Model in Applied Industrial Organization*, 3 INT'L J. ECON. BUS. 83, 85–87 (1996); Gregory J. Werden & Luke M. Froeb, *The Effects of Mergers in Differentiated Products Industries: Logit Demand and Merger Policy*, 10 J.L. ECON. & ORG. 407 (1994).

[39] Formally, the IIA property is that the ratio of the probabilities of any two choices is independent of the presence or absence of other alternative choices.

[40] Robert D. Willig, *Merger Analysis, Industrial Organization Theory, and Merger Guidelines*, BROOKINGS PAPERS ON ECONOMIC ACTIVITY, MICROECONOMICS 281, 299–305 (1991) argued that the logit model, with its IIA property, provides an appropriate benchmark and used the logit model to motivate reliance on market shares in the analysis of differentiated products mergers. Willig's view appears to be reflected in U.S. Department of Justice & Federal Trade Commission, Horizontal Merger Guidelines § 2.211, *reprinted in* 4 TRADE REG. REP. (CCH) ¶ 13,104 (1992, rev. ed. 1997):

> The market concentration measures articulated in Section 1 [of the Guidelines] may help assess the extent of the likely competitive effect from a unilateral price elevation by the merged firm notwithstanding the fact that the affected products are differentiated. The market concentration measures provide a measure of this effect if each product's market share is

usefully defines what it means for a group of brands to be equally close substitutes for each other. One justification for this definition is that the IIA property implies that the all cross elasticities of demand, with respect to any one price, are exactly the same. The equality of cross elasticities follows directly from substitution being proportionate to relative shares.

Economists' have long noted that the IIA property is not likely to hold in the real world. It is basically always true that a model not imposing the IIA property fits a real-world industry better than the logit model.[41] Nevertheless, we find the logit model very useful, at least as a starting point for the analysis of differentiated products mergers. Until reliable contrary evidence is uncovered, it sensible to presume that the products of the merging firms are neither especially close nor especially distant substitutes, which means that the IIA property holds approximately. And merger simulation using the logit model provides a highly useful initial indication of the potential consumer injury from a differentiated products merger.

To illustrate how logit merger simulations can enhance the focus and accuracy of a merger investigation, we consider the acquisition of Pripps Ringnes by Carlsberg, which already owned Falcon. The Swedish Competition Authority allowed the acquisition to proceed with divestitures.[42] We understand that a focus of the Authority's competitive concerns was Class II beer (folköl) sold in retail food stores (Class II beer), including beer with alcohol contents of 2.8% and 3.5%. We surmise that the Authority quickly began to pursue the possibility that a relevant market was Class II beer and quickly learned the

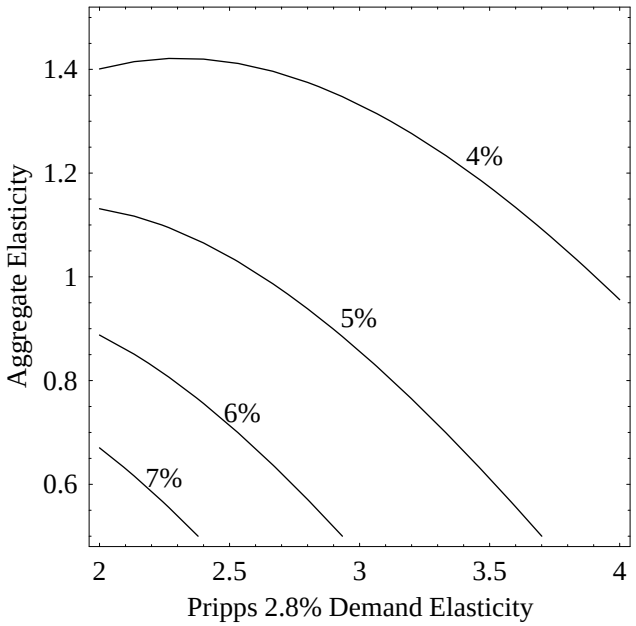
> reflective of not only its relative appeal as a first choice to consumers of the merging firms products but also its relative appeal as a second choice, and hence as a competitive constraint to the first choice. Where this circumstance holds, market concentration data fall outside the safeharbor regions of Section 1.5, and the merging firms have a combined market share of at least thirty-five percent, the Agency will presume that a significant share of sales in the market are accounted for by consumers who regard the products of the merging firms as their first and second choices.

[41] The logit model restricts substitution patterns only for the brands included in a simulation. Thus, the narrower the range of included brands, the less restrictive the logit model is. And since excluding brands typically is of little consequence to the price-increase predictions, the range of included brands may be quite narrow.

[42] Our discussion is based on information contained in the Authority's December 12, 2000 decision on the merger, as translated for us by Karl Lundvall, and in an on-line description of the beer industry in Sweden: http://www.xs4all.nl/~patto1ro/swedintr.htm.

average retail prices and shares for Class II beer.[43]

The logit model has two demand parameters, and with knowledge of neither, we consider a range of values. If Class II beer is a relevant market, the aggregate elasticity of demand for it must be less than the critical elasticity of demand, and for price-cost margins typical of U.S. manufacturing, the critical elasticity would be roughly 1.5. Thus, we consider values of 0.5 to 1.5. Given prices, shares, and an aggregate elasticity, completing the model requires selecting a value for the one remaining demand parameter, and that may be done by fixing the value for any of the brand-level elasticities of demand. We fix the elasticity of demand for Pripps 2.8% alcohol beer,[44] and consider elasticity values from 2.0 to 4.0. The figure below presents the results of the simulations in the form of a contour plot of the weighted average price increase for all Class II beer.



[43] As is typical with differentiated consumer products, the readily available price and share data are those for the retail level. To simulate competition among brewers, therefore, requires a model relating the retail and wholesale levels. The need for that model is acute in this case because of the differential tax treatment of 2.8% and 3.5% beer. We assume the simplest model: Let w_i be the wholesale price of brand i in SEK/liter and r_i its retail price. Assume retailers set prices by marking up the wholesale price by a fixed proportion s, then add any alcohol taxes. With a VAT of 12%, $r_i = (.12 + s)w_i$ for 2.8% beer, and for 3.5% beer, which has an added alcohol tax of 5.145 SEK/liter, $r_i = (.12 + s)w_i + 5.145$. Lacking any direct information, we assume $s = 1.3$, which is roughly consistent with margins for supermarkets in the United States.

[44] We aggregate all brands with the same brewer and alcohol content. Hence, the Pripps 2.8% demand elasticity is that for an aggregate of all Pripps 2.8% brands.

Merger simulation would have enhanced the accuracy of the evaluation of the acquisition by indicating a range of likely price increases: 4–7% for the weighted average of Class II beer prices at the wholesale level. This is a relatively narrow range, considering the wide range of demand parameters, and further analysis could have narrowed the range of price increases by narrowing the ranges of the demand parameters. Merger simulation also would have enhanced the accuracy of the evaluation of the acquisition by indicating not merely that the substitutability of other classes of beer was important, but also the likely impact on price increases of varying degrees of substitutability, as reflected in the aggregate elasticity. Of course, the price-increase predictions from merger simulations never should be taken as definitive, but rather only as a useful guide to the magnitude of the likely anticompetitive effects of a merger.

The simulations also would have added focus to the investigation by indicating what it would have had to reveal to alleviate competitive concerns. Findings that could have significantly alleviated concerns are: (1) that all Class II beers were viewed by consumers as essentially fungible, (2) that the Pripps and Carlsberg brands of Class II beer were relatively distant substitutes in the minds of consumers, or (3) that the acquisition would have produced large enough reductions in marginal cost to offset the price increases.

The merger simulation also would have focused the investigation on whether the basic assumptions of the model are appropriate. Evidence could have been amassed on the relevance of the differentiated product Bertrand model by assessing the importance of brands, determining whether price was the key strategic variable for competition, and indicating whether the intensity of existing competition, as reflected in price-cost margins, was consistent with the Bertrand model. If the investigation provided strong support for the model, that would have greatly enhanced the persuasiveness of the simulation analysis in court.

As the investigation proceeded, it may also have been possible to estimate the relevant demand elasticities. If so, the simulation analysis could have been refined significantly, and a model of demand other than simple logit might have been used.[45]

[45] For an analysis of two U.K. beer mergers using econometrics-based simulation, see Joris Pinkse & Margaret E. Slade, Mergers, Brand Competition, and the Price of a Pint (University of British Columbia working paper April 2002), *available at* http://www.econ.ubc.ca/slade/beer.pdf.

Among the options are generalizations of the logit model.[46] Econometrics-based merger simulation substantially increases the accuracy and persuasiveness of merger analysis by basing price-increase predictions directly on the underlying data from which the relevant demand elasticities are estimated.

Merger simulation also increases the focus and accuracy of the analysis of differentiated products merger is other ways. It provides a mechanism for explicitly trading off a reduction in competition against cost reductions from merger synergies. And it provides a mechanism for evaluating possible remedies, most notably the divestiture of particular brands. If brands are not all equally good substitutes for each other, or if there are synergies from the combination of just some of the merging firms' brands, simulation can enhance the accuracy of merger analysis by indicating the best remedy.[47]

An important limitation of merger simulation with differentiated products is that price-increase predictions are sensitive to the functional form for demand. Conventional functional forms all impose both particular rates at which each product's demand becomes more elastic as its price is increased, and idiosyncratic responses of cross elasticities to price changes.[48] A direct consequence is that the functional form of demand

---

[46] One generalization is the nested logit model, in which "nests" are placed around brands that are especially close substitutes. *See, e.g.*, MOSHE BEN-AKIVA & STEVEN R. LERMAN, DISCRETE CHOICE ANALYSIS: THEORY AND APPLICATION TO TRAVEL DEMAND ch. 10 (1985); JEFFREY A. DUBIN, STUDIES IN CONSUMER DEMAND—ECONOMETRIC METHODS APPLIED TO MARKET DATA ch. 6–7 (1998). Generalizations popular in academic research focus on brand characteristics. *See, e.g.*, Steven T. Berry, James Levinsohn & Ariel Pakes, *Automobile Prices in Market Equilibrium*, 63 ECONOMETRICA 841 (1995); Timothy F. Bresnahan, Scott Stern & Manuel Trajtenberg, *Market Segmentation and the Sources of Rents from Innovation: Personal Computers in the Late 1980s*, 28 RAND J. ECON. S17 (1997); Aviv Nevo, *A Practitioner's Guide to Estimation of Random-Coefficients Logit Models of Demand*, 9 J. ECON. & MGMT. STRATEGY 513 (2000).

[47] *See* Jith Jayaratne & Carl Shapiro, *Simulating Partial Asset Divestitures to "Fix" Mergers*, 7 INT'L J. ECON. BUS. 179 (2000). It is also possible to incorporate any effects of remedies on costs. For example, it is possible to model royalty payments from one competitor to another that could result from a licensing arrangement. While probably never an appropriate remedy, it is easy to incorporate pricing limitations in a simulation.

[48] Most conspicuously, with constant elasticity demand, all own and cross elasticities are invariant to prices.

substantially determines the magnitude of price increases from a merger. Of the demand forms in common use, linear and logit demand yield the smallest price increases. Two other commonly used functional forms, constant elasticity and AIDS demand,[49] typically yield price increases that are *at least* several times those with linear or logit demand.[50] The same properties that cause different demand forms to yield very different price increase also cause them to yield very different pass-through rates for marginal-cost reductions.[51]

The dependence of merger simulation on the functional form of demand suggests the desirability of using calibrated economic models in a manner that makes them insensitive to the functional form of demand. This is done by computing the compensating marginal cost reductions (CMCRs), i.e., those that exactly offset the price-increasing effects of a merger. CMCRs do not depend on the functional form of demand for the simple reason that the equilibrium prices and quantities post merger are precisely the same as those pre merger. Using the same inputs as merger simulation—prices, shares, and demand elasticities—it is relatively simple to compute the CMCRs for a differentiated products merger.[52] If merger synergies appear likely to reduce the merging firms cost as much as the CMCRs, it follows that the merger is unlikely to harm consumers. And if merger synergies clearly fall well short of those necessary to prevent price increase, it follows that significant price increase are likely.[53]

---

[49] On the AIDS model, see Angus Deaton & John Muellbauer, *An Almost Ideal Demand System*, 70 AM. ECON. REV. 312 (1980). For discussions of merger simulation using AIDS demand, see Crooke et al., *supra* note 34; Roy J. Epstein & Daniel L. Rubinfeld, *Merger Simulation: A Simplified Approach with New Applications*, 69 ANTITRUST L.J. 833 (2002); Hausman & Leonard, *supra* note 30.

[50] *See* Crooke et al., *supra* note 34.

[51] *See* Gregory J. Werden, Luke M. Froeb & Steven Tschantz, The Effects of Merger Synergies on Consumers of Differentiated Products (unpublished paper 2001).

[52] *See* Gregory J. Werden, *A Robust Test for Consumer Welfare Enhancing Mergers among Sellers of Differentiated Products*, 44 J. INDUS. ECON. 409 (1996).

[53] A similar, but simpler, analysis can be done for mergers in homogeneous goods industries. *See* Gregory J. Werden & Luke M. Froeb, *A Robust Test for Consumer Welfare Enhancing Mergers among Sellers of a Homogeneous Product*, 58 ECON. LETTERS 367 (1998).

Conclusions

Calibrated economic models provide concrete, quantitative analyses of market delineation and the competitive effects of mergers. These analyses are firmly grounded in the facts of the case and based on well-accepted models of monopoly and oligopoly. Their use significantly enhances the focus, accuracy, and persuasiveness of merger analysis in many ways.

Nevertheless, some lawyers are reluctant to rely on calibrated economic models, especially in court. The main reason appears to be a belief that such analyses cannot be adequately understood by judges and thus appear as a black box. Our view is almost precisely the opposite. Expert analysis based on calibrated economic models is a black box only if presented in a highly summary, and clearly inappropriate, fashion. When calibrated economic models are properly used and presented, they make clear how an expert's conclusions follow from the facts of the case. Economic models are built on assumptions, which should be stated explicitly. Once explicitly stated, assumptions can be attacked and defended largely on the basis of the factual record in the case. The use of calibrated economic models therefore makes the battle of the experts into what it should be—a debate over links in a chain of economic logic connecting established facts to ultimate conclusions. Useful economic analysis identifies the links that really matter and explains them in terms judges can comprehend.